Debt Sustainability Condition ±Mathematical Version

The famous debt sustainability condition is often written mathematically as follows:

¿ I nš / I š · š p L :˜š F •š; I nš? Ú / I š? Ú · š? Ú p E sš F €š / I š · š F ¿t š / I š · š

SIMPLE ANALYTICS:

Public Debt-to-GDP ratio will rise if:

˜š P •š

sš F €š P Ù

Debt sustainability requires:

¿ I nš / I š · š p L Ù

If debt monetization is pursued ¿*ç P r, it will help reduce the debt burden. However, debt monetization may result in higher inflation. If we rule out debt monetization (¿*ç L r), then debt sustainability requires:

F I sš F €š / I š · š p L :˜š F •š; I nš? Ú / I š? Ú · š? Ú p

or, !0ˆ^ y.ìÍ å y.¿1/ „˜CW •, à Kä a''ëó ñ I„5qp R0 ÄÓ a''ëó ñ I„5qp p† à >@à y.¿1/ •“ à

I €š F sš / I š · š p L :˜š F •š; I nš? Ú / I š? Ú · š?

)ç: Government Purchases

6ç: Taxes net of Transfers (6ç L 6 = T A F 6 N = J O) A N O

)ç F 6ç: Primary Budget Deficit (note:)ç P 6ç implies a primary budget deficit)

$ç: Government Debt in Period t

DERIVATION

Reference Macroeconomics (2nd Edition) by R. Glenn Hubbard and Anthony Patrick O'Brien (Publisher: Pearson)

Standard budget deficit is given by:

&ç L E$ç?5 E)ç F 6ç

To capture the role of seignorage (essentially represents a transfer of wealth from individuals holding money to the government), we extend the above equation to include change in monetary base:

&ç L E$ç?5 E)ç F 6ç F ¿*ç

Note that the change in government debt is given by:

¿nš L nš F nš?Ú L pš L •š nš?Ú E sš F €š F ¿t š

Divide both sides by nominal GDP and rearrange to get

nš / |š·š L :ÚE •š; nš?Ú / |š·š E sš / |š·š F €š / |š·š F ¿t š / |š·š

Modify pervious equation as follows (multiply and divide the first term on the right hand side by |š?Ú·š?Ú):

nš / |š·š L :ÚE •š; l |š?Ú·š?Ú / |š·š p l nš?Ú / |š?Ú·š?Ú p E sš / |š·š F €š / |š·š F ¿t š / |š·š

Note the following condition,

l |š·š / |š?Ú·š?Ú p L |š / |š?Ú ·š / ·š?Ú L :ÚE Êš; :ÚE •š;

l |š·š / |š?Ú·š?Ú p L :ÚE Êš; :ÚE •š; N ÚE Êš E •š

Using the approximation

l |š·š / |š?Ú·š?Ú p N ÚE Êš E •š

We get:

nš / |š·š L l ÚE •š / ÚE Êš E •š p l nš?Ú / |š?Ú·š?Ú p E sš / |š·š F €š / |š·š F ¿t š / |š·š

Another useful approximation:

l ÚE •š / ÚE Êš E • š p N ÚE •š F Êš F • š

Sα

nš / l š · ·š L :ÚE •š F Êš F • š; l nš? Ú / l š? Ú · ·š? Ú p E sš / l š · ·š F €š / l š · ·š F ¿t š / l š · ·š

Rearrange terms to get: